A complete axiomatization of timed bisimulation for

- The co u ntab e ordina s $(\omega\ \ , +,\ \ )$–

$$
\begin{array}{lrcl}
\text{S} & E+F & = & F+E \\
\text{S} & E+(F+G) & = & (E+F)+G \\
\text{S} & E+E & = & E \\
\text{S} & E+\mathbf{0} & = & E \\
\\
\text{TD} & \varepsilon(t).(E+F) & = & \varepsilon(t).E+\varepsilon(t).F \\
\text{TA} & \varepsilon(t+u).E & = & \varepsilon(t).\varepsilon(u).E \\
\text{T} & \varepsilon(\ ).E & = & E \\
\\
\text{R} & \mathbf{fix}(x=E) & = & E\{\mathbf{fix}(x=E)/x\}
\end{array}
$$

R If $F = E\{F/x\}$ then $F = \mathbf{fix}(x = E)$ provided x is action guarded in E

FIG RE – The axio syste $\mathcal{G}$

$$
\begin{array}{lrcl}
\text{MP} & \tau.E+\varepsilon(c).F & = & \tau.E \\
\text{AP} & a.E+\varepsilon(t).a.E & = & a.E \\
\text{NP} & \varepsilon(t).\mathbf{0} & = & \mathbf{0}
\end{array}
$$

FIG RE – The axio syste $\mathcal{F}$ is $\mathcal{G}$ p s MP AP and NP

$$
\begin{array}{lrcl}
\text{MP} & \tau.E+\varepsilon(c).F & = & \tau.E \\
\text{P} & E+\varepsilon(t).E & = & E
\end{array}
$$

FIG RE – The axio syste $\mathcal{E}$ is $\mathcal{G}$ p s MP and P

Then $E \sim F$ iff for all vectors $P = P\ ,\ldots,P_m$ $E\{P/x\} \sim F\{P/x\}$.

PROPOSITION THEOREM 5 – *Timed bisimulation equivalence forms a congruence over* **TC**.

In the re ainder of this paper we sha present a co p ete axio atization of $\sim$ over **TC**–

3 Axiomatization and soundness

In vario s eq ationa aws were proved to ho d for ang Yi s ti ed CCS od o ti ed bisi ation eq iva ence and in a set of s ch axio s was shown to be co p ete over the ang age of rec rsion free **TC** processes with de ays fro the ti e do ain of the positive rea s e sha now present an axio atization which wi be proven co p ete for $\sim$ over the who e of **TC** i e co p ete for reg ar process expressions with action g arded rec rsion–The detai ed proof of co p eteness occ pies Section of this paper–

ang s axio atization for rec rsion free **TC** processes is given by the axio syste $\mathcal{F}$ in Fig res and – O r axio atization for reg ar **TC** process expressions is given by the axio syste $\mathcal{E}$ in b – d e – x – 5

4 Completeness

In this section, we sha present the proof of co p eteness of the set of aws $\mathcal{E}$ over **TC**–The structure of the proof of this resu t wi fo ow c ose y the ost beautifu argu ents used by Mi ner in , to prove the co p eteness of the axio a tizations for strong bisi u ation and observationa congruence over regu ar CCS processes–

The structure of the co p eteness proof wi be as fo ows   rst of a , we sha show that every **TC** expression E provab y satis es a certain kind of equation set–This is what Mi ner ca s the *Equational Characterization Theorem*– Next, we sha show that if $E \sim F$ and E provab y satis es an equation set, whi e F provab y satis es another equation set, then both E and F provab y satisfy a co on equation set– Fina y, we show that whenever two **TC** expressions provab y satisfy the sa e equation set, then $\mathcal{E}$ proves that they are equa –

DEFINITION– *An* equation set $x = E$ *is a finite non-empty sequence of declarations* $x\ = E\ , \ldots, x_n = E_n$*, where the* x_i*s are pairwise distinct variables, and the* E_i*s are* **TC** *expressions.*

A vector $F = F\ \ldots F_n$ satis es $x = E$ *iff* $\forall i \,.\, F_i \sim E_i\{F/x\}$.

For an equational theory $\mathcal{T}$*, a vector* $F = F\ \ldots F_n$ $\mathcal{T}$-provab y satis es $x = E$ *iff* $\forall i \,.\, \mathcal{T} \vdash F_i = E_i\{F/x\}$.

An expression E *(*$\mathcal{T}$*-provably) satisfies* $x = F$ *iff we can find a vector* E *which (*$\mathcal{T}$*-provably) satisfies* $x = F$ *and* $E \sim E$ *(*$\mathcal{T} \vdash E = E$ *).*

We refer to x *as the leading variable of the equation set* $x = F$.

For exa p e, the equation set

$$x\ = \varepsilon($$

$$= E$$

and

$$\begin{aligned}\mathcal{G} \vdash E \\ &= F\,\{E/w\} \\ &= H\,\{F/x\}\{E/w\} \\ &= H\,\{E/w\}\{F\{E/w\}/x\} && \text{Propn 5.–} \\ &= H\,\{E/w\}\{E/x\} \\ &= H\,\{E/x\} && w \notin \mathrm{fv} H\end{aligned}$$

and so

$$\begin{aligned}\mathcal{G} \vdash E_i \\ &= F_i\{E/w\} \\ &= H_i\{F/x\}\{E/w\} \\ &= H_i\{E/w\}\{F\{E/w\}/x\} && \text{Propn 5.–} \\ &= H_i\{E/w\}\{E/x\} \\ &= H_i\{H\,\{E/x\}/w\}\{E/x\} && \text{above} \\ &= H_i\{H\,/w\}\{E/x\} && \text{Propn 5.–5.} \\ &= G_i\{E/x\}\end{aligned}$$

Thus we have found a standard $x = G$ which E $\mathcal{G}$ provab y satis es– □

Theore shows that every expression E in **TC** $\mathcal{G}$ provab y satis es a standard equation set $x = G$– The second stepping stone towards the pro ised co p ete ness theore is a resu t showing that if $E \sim F$, where F $\mathcal{G}$ provab y satis es a standard equation set $y = H$, then there re re re re t

Thus each s… and of $G_{ii'}\{H/z\}$ can be absorbed into $H_{ii'}$, and by S S

$$\mathcal{E} \vdash H_{ii'} = H_{ii'} + G_{ii'}\{H/z\}$$

We now show that the converse also holds, namely that $H_{ii'}$ can be absorbed into $G_{ii'}\{H/z\}$. To this end, by and , it is sufficient to prove that each s… and of $F_i\{E/x\}$ can be absorbed into $G_{ii'}\{H/z\}$. Again, we distinguish three cases depending on the form the s… and takes.

For any $i \mathcal{R} i'$ and $j \in J_i$, either

- $t_j \leq u_k$ for every $k \in K_i$, or
- there exists $k \in K_i$ such that $t_j > u_k$.

We proceed to show that in either case

$$\mathcal{E} \vdash G_{ii'}\{H/z\} = G_{ii'}\{H/$$

5

$$
\begin{aligned}
&= H_{ii'} + G_{ii'}\{H/z\} \\
&= E_i + G_{ii'}\{H/z\} \\
&= F_i\{E/x\} + G_{ii'}\{H/z\} \\
&= G_{ii'}\{H/z\}
\end{aligned}
$$

Thus H $\mathcal{E}$ provab y satis es $z = G$ and $\mathcal{E} \vdash E = E \; = H$ so E $\mathcal{E}$ provab y sat is es $z = G$–Si i ar y E' $\mathcal{E}$ provab y satis es $z = G$– □

The na ingredient of the proof of co p eteness is a resu t showing that every standard equation set has a unique so ution up to provab e equa ity–

THEOREM NIQ E SOL TION – *If $x = H$ is a standard equation set, then there is a* **TC** *expression E which $\mathcal{E}$-provably satisfies it. Moreover, if another* **TC**

$$= \sum_{i\in I} \varepsilon(t_i).\mu_i.P_i + \varepsilon(t+t_i).\mu_i.P_i \qquad t+(u-t)=u$$

$$= \sum_{i\in I} \varepsilon(t_i).\mu_i.P_i + \varepsilon(t).\varepsilon(t_i).\mu_i.P_i \qquad \text{TA}$$

$$= \sum_{i\in I} \varepsilon(t_i).\mu_i.P_i + \sum_{i\in I} \varepsilon(t).\varepsilon(t_i).\mu_i.P_i \qquad \text{S , SF}$$

$$= \sum_{i\in I} \varepsilon(t_i).\mu_i.P_i + \varepsilon(t).\sum_{i\in I} \varepsilon(t_i).\mu_i.P_i \qquad \text{TD, NP}$$

$$= P + \varepsilon(t).P$$

Thus $\mathcal{F}$ can show any c osed instantiation of axio P – □

Note that throughout the above proof we have been careful not to assu e that the onoida operation + on the ti e do ain is co utative–A though this is true for ost of the exa p es of ti e do ain one encounters in the iterature, it does not ho d for, e–g–, the ti e do ain of the countab e ordina s (ω , +,)–

6 Concluding remarks

In this paper, we have presented a co p ete axio atization of ti ed bisi u ation equiva ence over open ter s with nite state recursion in a genera ization of the regu ar subca cu us of ang s ti ed CCS–Our inference syste for ti ed bisi u ation equiva ence is obtained by co bining an i proved version of ang s co p ete axio atization for nite trees F with standard aws for recursive y de ned processes– The proof of co p eteness of the proposed axio atization uses an adaptation of Mi ner s c assic argu ents presented in , –

In t pea

Holmer, K.G. Larsen, and Wang Yi. Deciding properties of regular real timed processes. Report [illegible], Institut for Electronic Systems, Department of Mathematics and Computer Science, Aalborg University Centre, [illegible]. An extended abstract appeared in the *Proceedings of CAV '91*.